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LV NETWORK TOPOLOGY AND PARAMETER ESTIMATION WITH

Deliverable D3.4

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Deliverable D3.4

LV NETWORK TOPOLOGY AND PARAMETER ESTIMATION WITH HIGH DATA AVAILABILITY

Version 1.0

Organisation: CERTH

Main authors: Stefanos Petridis, Orestis Blanas, Dimitrios Rakopoulos, Fotis Stergiopoulos, Nikolaos Nikolopoulos, Spyridon Voutetakis, Eleftherios Andritsakis, Nikolaos Tagkoulis

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DELIVERABLE 3.4 – VERSION 1.0

WORK PACKAGE N° 3 – Tool for low voltage grid modelling

TASK N° 3.4 – LV network topology and parameter estimation with high data availability

Nature of the deliverable		
R	Document, report (excluding the periodic and final reports)	
DEM	Demonstrator, pilot, prototype, plan designs	
DEC	Websites, patents filing, press & media actions, videos, etc.	
OTHER	Software, technical diagram, etc.	X

Dissemination Level		
PU	Public, fully open, e.g. web	
CO	Confidential, restricted under conditions set out in Model Grant Agreement	X
CI	Classified, information as referred to in Commission Decision 2001/844/EC	

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Executive summary

This report presents the work towards addressing the requirements for Task 3.4 of Work Package 3. The objective of Task 3.4 is to create and propose algorithms that operate in high data availability scenarios. These algorithms will produce the outputs needed for the next Work Packages. In high data availability scenarios, there is an extensive set of measurements available. So, the main goal of the algorithms is to verify these measurements and produce a more accurate grid model. The activities from M9 to M14 led indeed to the fulfillment of this objective.

First, the available data for the grid components were organized in a structured way in order to be used as input for the algorithms to be developed. The equations that model each component were also developed for each algorithm separately. The two main algorithms developed are the Power Flow Algorithm (PFA) and the State Estimation Algorithm (SEA). Both were written and run in Python to test their validity. Aside from calculating the state of the network, they contain an additional feature each. The PFA is capable of finding the optimal tap settings of all transformers on the network, thus ensuring a voltage profile close to the nominal one throughout the network. The SEA is capable of detecting and discarding bad measurements, which can be e.g. the result of malfunctioning meters or cyberattacks. Finally, flowcharts were developed for all algorithms to be given as input to Task 3.5, which is responsible for developing a software implementing those algorithms.

Task 3.4's result succeeds in providing the following functionalities, which had been indicated-targeted by the tentative, non-binding set of Use Cases (UCs) proposed within D3.1 for the Tasks 3.2 – 3.4:

- Unbalanced 3-phase load flow (UC 3.01, Transversal)
- State estimator (UC 3.02, Transversal)
- Customer phase estimator (UC 3.13)
- Loading Scenarios (UC 3.19)
- Transformer tap position estimator (UC 3.21)
- Loading imbalance estimator (UC 3.31)

The first two functionalities are implemented by the PFA and SEA, respectively. The input of the PFA can be measured loads or load profiles as required by UC 3.19. The first output vector of the SEA, which can contain tap positions, satisfies UC 3.21, while UCs 3.13 and 3.31, are satisfied by the second output vector of SEA, containing the consumed power of loads for each bus.
It must be noted that the only functionality (UC 3.14, tentatively suggested by D3.1) which is not supported by the herewith developed algorithms is the cable data estimator. This is because the cable data are necessary for the specific algorithms to work and so are considered valid. Theoretically, cable data can be validated through the SEA, but the number of measurements needed to validate the data for both the measurements and the cables is extremely high. However, apart from cable data verification, all suggested use cases of Task 3.1 are tackled.

Use Case	Transv.	Low	Medium	High
UC3.01 Unbalanced 3P load flow	•			
UC3.02 State estimator	•			
UC3.03 Generate grid model	•			
UC3.04 CIM model generation	•			
UC3.11 Cable interconnections		•	•	
UC3.12 Customer location estimator – Node		•	•	
UC3.13 Customer location estimator – Phase		•	•	•
UC3.14 Parameter estimator – Cable		•	•	•
UC3.15 Parameter estimator – Transformer		•		
UC3.16 Data extrapolation – Load PQ profiles		•		
UC3.17 Data extrapolation – Transformer PQ profiles		•		
UC3.18 Data extrapolation – PV generation profile		•		
UC3.19 Loading Scenarios		•	•	•
UC3.21 Parameter estimator – Transformer Tap pos.			•	•
UC3.31 Data extrapolation – Imbalance in 3P loads				•

Figure 1: Tentatively proposed Use Cases in Task 3.1 addressed by Task 3.4

List of abbreviations

Abbreviation	Full name
DSO	Distribution System Operators
LV	Low Voltage
TSO	Transmission System Operations
PFA	Power Flow Algorithm
TOA	Tap Optimisation Algorithm
SEA	State Estimation Algorithm

Partners short names

CIRCE: FUNDACIÓN CIRCE CENTRO DE INVESTIGACIÓN DE RECURSOS Y CONSUMOS ENERGÉTICOS

RDN: CENTRO DE INVESTIGACAO EM ENERGIA REN - STATE GRID SA – R&D NESTER

CERTH: ETHNIKO KENTRO EREVNAS KAI TECHNOLOGIKIS ANAPTYXIS

DTU: DANMARKS TEKNISKE UNIVERSITET

CARTIF: FUNDACION CARTIF

ATOS: ATOS SPAIN SA

ATOS IT: ATOS IT SOLUTIONS AND SERVICES IBERIA SL (LTP ATOS)

ORES: OPERATEUR DE RESEAUX D'ENERGIES

CUERVA: MONTAJES ELECTRICOS CUERVA S.L.

TT: TURNING TABLES S.L. (LTP CUERVA)

EQY: EUROQUALITY SARL

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1. INTRODUCTION

In order to properly control and protect the power system, its state must be known so that appropriate actions are taken. Its state is defined as the magnitude and phase of the 3 voltages of each bus. Depending on the available data and measurements from the power system, the proper method to calculate the voltages can be selected. The two main methods are the standard power flow algorithm (PFA) and the standard state estimation algorithm (SEA). Both are discussed in detail in chapter 9 and chapter 15 of [1], respectively. Of course, the appropriate algorithm must be applied on the transmission and the distribution level of the power system, achieving a complete view of the state variables of the entire system. Since the transmission level is considered balanced in many cases, the standard algorithms are effective when it comes to applying them on transmission systems. On the other hand, a lot of research has been done on modelling distribution systems and their components for use in PFA and SEA. Such research is necessary as distribution networks present characteristics that differ from those of transmission networks. These characteristics include the presence of distributed generation, which is used increasingly due to its lower environmental impact, a radial network structure, single phase or unbalanced consumer loads and high resistance to reactance ratio on lines [2]. For these reasons, the traditional algorithms, like the standard PFA and the standard SEA, that assume a balanced network, are insufficient. In addition to the imbalances present, the number of iterations and convergence of PFA and SEA are further affected by the characteristics of the distribution network to be studied, the method used to construct and solve the system equations and the reference frame and transformer model used.

Regarding the PFA, the standard Newton-Raphson Algorithm used for transmission systems cannot be used in many cases due to the topology of the distribution systems [2]. A way to tackle this problem is to take advantage of the radial structure of distribution networks through a Backward-Forward Sweep Algorithm (BFSA). It involves separating the original system of equations into two separate systems and solve one, using the last results of the other, until convergence is achieved. A BFSA is utilized in [3], along with an abc reference frame, which may cause a failure to converge, while [4] uses BFSA along with an admittance matrix to formulate the problem, but in case any system component is altered this matrix must be rebuilt, raising the complexity. Finally, a dq0 reference frame was implemented in [5], but did not include transformer modelling.

The most thorough algorithm for power flow was found in [6], [7] and [8]. It includes the use of a BFSA, in combination with the αβ0 reference frame, which allows modelling both transformers and regulators in a new and unified way similar to transmission lines. The system equations are written using the node incidence matrix instead of the admittance matrix of the power system. These features ensure convergence, even if the initial voltage profile estimate is not optimal. CERTH adopted the transformer modelling and the BFSA presented in [6]-[8], but modified the BFSA to solve the tree formed by the distribution network level by level, which leads to very short running times, even on systems with a great number of buses. Though the algorithm was developed in Python, this modification can be used in any programming language and as such is extremely useful. The short running time resulting from the algorithm modification allowed tap optimization to be studied with an exhaustive search of all possible tap combinations. Exhaustive search always succeeds in finding the global optimum, unlike other ways to determine the optimal taps, like the one found in [9]. Though [9] succeeds in reducing the time required for tap optimization by using reinforcement learning to produce solutions, sometimes the global optimum is missed and a local optimum is found instead.

Regarding the SEA, there are two possible methods when it comes to balanced power systems. The traditional one presented in chapter 15 of [1] and a novel one presented in [10]. In [10], in order to estimate the state of the power system, the system is considered as the sum of branches, that are themselves small subsystems. Then, each subsystem is solved and its state is estimated individually and the results are used in the state estimation of the following subsystems, in order to reduce the complexity and running time of the algorithm. However, this solution lacks in accuracy compared to the standard algorithm of [1] and is more difficult to implement in unbalanced networks. Thus, it was not chosen. The SEA of [1] must be properly modified to accommodate for the imbalances of an unbalanced power system. The main differences from the traditional balanced case SEA are the relationships between each measurement and the state variables, as well as the modelling of the transformers. Both are addressed by [11]. However, in [11] only step-down transformers connections are mentioned. Both step-down and step-up transformer connections are discussed in [4]. Utilizing the relationships given in [4] and [11], a SEA for unbalanced power systems can be created using the one of balanced systems as a basis. Finally, after the state estimation of a power system is done, bad data detection is integrated in the SEA and can be used to filter out false measurements as presented in chapter 15 of [1].

In conclusion, CERTH will utilize the high data availability of its assigned scenario to propose both a PFA and a SEA, that are applicable for unbalanced power systems. The PFA has been coded using a procedure that minimizes its running time and allows for tap optimization, while a process to filter bad data is derived from the results of the SEA. The combination of these tools makes up a very useful package for any distribution system operator.

2. MATHEMATIC SYMBOLISMS & UNITS

Complex numbers are represented with a line above them (accent):	$\overline{\mathrm{N}}$
Matrices containing only real numbers are represented with bold letters:	$\mathbf{M}$
Matrices containing complex numbers are represented with bold letters and an accent:	$\overline{\mathbf{M}}$
Matrices can contain numbers, other matrices or both	
Vectors are n x 1 matrices.	
Vectors in the abc system are represented without a superscript:	$\mathbf{vector}$ or $\overline{\mathbf{vector}}$
Vectors in the αβ0 system are represented with the αβ0 superscript:	$\mathbf{vector}^{\alpha\beta0}$ or $\overline{\mathbf{vector}}^{\alpha\beta0}$
All non-vectors matrices are in the abc system.	
The identity matrix is represented by:	$\mathbf{Id}$
The three-phase 3 x 1 line-to-neutral voltage vector at bus i is represented as:	$\overline{\mathbf{V}}_{\mathbf{i}}$
The three-phase 3 x 1 current vector at bus i with direction from i to j is represented as:	$\overline{\mathbf{I}}_{\mathbf{ij}}$
The three-phase 3 x 1 power vector at bus i with direction from i to j is represented as:	$\overline{\mathbf{S}}_{\mathbf{ij}} = \mathbf{P}_{\mathbf{ij}} + \mathbf{j} * \mathbf{Q}_{\mathbf{ij}}$

For all algorithms described in this document and their respective inputs and outputs:

- angles are in degrees (°)
- distances are in feet (ft)
- voltages are in Volts (V)
- currents are in Amperes (A)
- impedances (Z) are in Ohms per mile (Ω/mile) for distribution lines, and in per-unit with the rated power and voltages as bases for transformers.

- apparent, real and reactive powers are in Voltamperes (VA), Watts (W) and Voltampere reactive (VAR) respectively
- transformer and regulator ratios are in Volts per Volt (V/V)

When it is stated that a 3 x 1 complex vector $\begin{bmatrix} \bar{k}_a \\ \bar{k}_b \\ \bar{k}_c \end{bmatrix}$ is converted to a 6 x 1 vector, then it means it is converted to the vector $\begin{bmatrix} re(\bar{k}_a) \\ re(\bar{k}_b) \\ re(\bar{k}_c) \\ im(\bar{k}_a) \\ im(\bar{k}_b) \\ im(\bar{k}_c) \end{bmatrix}$.

When it is stated that 3 x 3 matrix $\bar{\mathbf{L}}$ is converted to a 6 x 6 matrix, then it means it is converted to matrix $\begin{bmatrix} re(\bar{\mathbf{L}}) & -im(\bar{\mathbf{L}}) \\ im(\bar{\mathbf{L}}) & re(\bar{\mathbf{L}}) \end{bmatrix}$.

Note: An equation of the form $\begin{bmatrix} \bar{k}_a \\ \bar{k}_b \\ \bar{k}_c \end{bmatrix} = \bar{\mathbf{L}} * \begin{bmatrix} \bar{m}_a \\ \bar{m}_b \\ \bar{m}_c \end{bmatrix}$ can be written in the form

$$\begin{bmatrix} re(\bar{k}_a) \\ re(\bar{k}_b) \\ re(\bar{k}_c) \\ im(\bar{k}_a) \\ im(\bar{k}_b) \\ im(\bar{k}_c) \end{bmatrix} = \begin{bmatrix} re(\bar{\mathbf{L}}) & -im(\bar{\mathbf{L}}) \\ im(\bar{\mathbf{L}}) & re(\bar{\mathbf{L}}) \end{bmatrix} * \begin{bmatrix} re(\bar{m}_a) \\ re(\bar{m}_b) \\ re(\bar{m}_c) \\ im(\bar{m}_a) \\ im(\bar{m}_b) \\ im(\bar{m}_c) \end{bmatrix}.$$

3. MODELLING

In order for the equations of both the PFA and the SEA to be constructed, components of the power system such as generators, loads, transformers, regulators and distribution lines must be properly modelled. Due to the PFA and the SEA being different, the modelling of certain components may differ for each algorithm. This is because each model is convenient for certain algorithms and inconvenient for others. However, all models used for a component are equivalent and can be derived from each other.

3.1. THE PER-UNIT SYSTEM

Three-phase transformers are one of the most important components of modern power systems as they allow voltage level changes with few losses. In general, if a power system has n transformers, then there are n+1 voltage levels in the power system. Each voltage level is equal to the rated transformer voltage on the corresponding side. Of course, each level can have a voltage equal to the voltage of another level. For example, a power system with 2 transformers may have the following 3 voltage levels: 220 V, 400 V, 220 V. However, each power system contains numerous transformers with a different voltage level on each side. Thus, there is a need to do complex calculations, due to the many different voltage levels present in the system. The per-unit system is used to simplify these calculations. In addition, the per-unit system makes the terminating criterion (usually the difference between the values of subsequent iterations) of many algorithms the same for every power system, regardless of the number of voltage levels present. This allows a unified approach for all power systems. In order to use the per-unit system each voltage, power, current and impedance is divided by a base quantity.

First, a base power S_{base} is selected for the whole power system. The base power is equal to the greatest rated power among the components of the power system under study.

Then, a base voltage V_{base} is selected for each part of the power system with a different voltage level. This means that n+1 base voltages are selected, if a power system has n transformers. A base voltage of a section can be equal to the base voltage of a different section of the power system. Each base voltage is equal to the rated voltage of the respective transformer side.

Using the base power of the system and the base voltages of each section defined by the transformers, the other base quantities for each section can be calculated by:

$$I_{base} = \frac{S_{base}}{V_{base}} \tag{1}$$

$$Z_{base} = \frac{V_{base}^2}{S_{base}} \tag{2}$$

Afterwards, each voltage, power, current and impedance is divided by its base quantity:

$$\overline{V}_{pu} = \frac{\overline{V}}{V_{base}} \tag{3}$$

$$\overline{S}_{pu} = \frac{\overline{S}}{S_{base}} \tag{4}$$

$$P_{pu} = \frac{P}{S_{base}} \tag{5}$$

$$Q_{pu} = \frac{Q}{S_{base}} \tag{6}$$

$$\overline{I}_{pu} = \frac{\overline{I}}{I_{base}} \tag{7}$$

$$\overline{Z}_{pu} = \frac{\overline{Z}}{Z_{base}} \tag{8}$$

These per-unit quantities are used for all equations in the following sections of this report. A per-unit quantity can be reverted to its normal value by multiplying it with its base. Many power system components, such as transformers, specify their impedance in the per-unit system, using their rated power and voltage as bases. If the base power or base voltage of a component differs from the base power or base voltage chosen for its section of the power system, its appropriate per-unit impedance for the given section can be calculated by:

$$\overline{Z}_{pu,new} = \frac{S_{base,new}}{S_{base,old}} * \frac{V_{base,old}^2}{V_{base,new}^2} * \overline{Z}_{pu,old} \tag{9}$$

All per-unit quantities have the unit pu (from per-unit).

3.2. THE αβ0 SYSTEM

Each voltage, power and current in an unbalanced power system with three phases is a 3 x 1 vector, where each vector element corresponds to one phase. For example, the voltage of bus i is:

$$\overline{\mathbf{V}}_{\mathbf{i}} = \begin{bmatrix} \overline{\mathrm{V}}_{\mathrm{i,a}} \\ \overline{\mathrm{V}}_{\mathrm{i,b}} \\ \overline{\mathrm{V}}_{\mathrm{i,c}} \end{bmatrix}$$

In order to ensure convergence of the PFA [6] and further simplify its equations, the αβ0 system is used alongside the per-unit system, instead of the abc system. If a 3x1 vector $\overline{\mathbf{vector}}$ is given in the abc system, it can be converted to the 3 x 1 αβ0 system vector $\overline{\mathbf{vector}}^{\boldsymbol{\alpha\beta0}}$ by:

$$\overline{\mathbf{vector}}^{\boldsymbol{\alpha\beta0}} = \mathbf{A}^{-1} * \overline{\mathbf{vector}} \qquad (10)$$

where

$$\mathbf{A} = \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & 0 & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

and $\mathbf{A}$ has the useful property:

$$\mathbf{A}^{-1} = \mathbf{A}^{\mathbf{T}} \qquad (11)$$

So, (10) becomes:

$$\overline{\mathbf{vector}}^{\boldsymbol{\alpha\beta0}} = \mathbf{A}^{\mathbf{T}} * \overline{\mathbf{vector}} \qquad (12)$$

Similarly, a 3 x 1 vector in the αβ0 system $\overline{\mathbf{vector}}^{\boldsymbol{\alpha\beta0}}$ can be converted to the 3 x 1 abc system vector $\overline{\mathbf{vector}}$ by:

$$\overline{\mathbf{vector}} = \mathbf{A} * \overline{\mathbf{vector}}^{\boldsymbol{\alpha\beta0}} \qquad (13)$$

The αβ0 system is used exclusively for the PFA and not for the SEA. The SEA uses the abc system, since after testing the αβ0 system, no improvement in its convergence was observed.

3.3. GENERATORS, LOADS AND BUSES

Generators convert mechanical into electrical energy in order to satisfy the energy demand of the customers. There are 2 types of generators depending on the control technique applied on them: Power-Voltage generators, PQ generators.

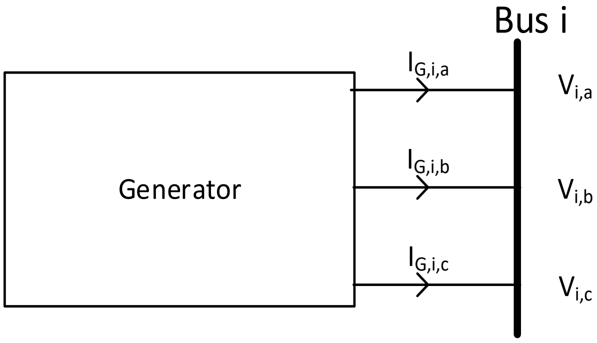

Figure 2: A generator connected to bus i

Loads represent customer consumption, capacitor banks or distributed generation and are connected in Y_g or Δ configuration. There are 3 types of loads: PQ loads, I loads, Z loads. For example, data centers, electric vehicles or DC motors are usually represented as PQ loads, street lighting and renewable energy sources are usually represented as I loads. Resistance heating loads and parasitic impedances of distribution are usually represented as Z loads.

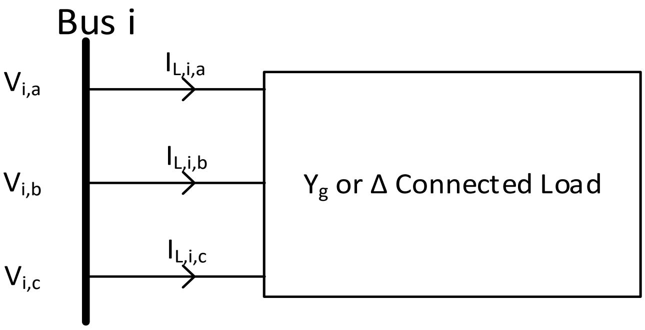

Figure 3: A load connected to bus i

Buses are the nodes of the power system. There are 3 types of buses: Slack buses, PV buses, PQ buses.

A bus with known voltage magnitude and angle is a slack bus. Each power system has only 1 slack bus, that is described by:

$$\bar{\mathbf{V}}_{\mathbf{i}} = \begin{bmatrix} \bar{V}_{i,a} \\ \bar{V}_{i,b} \\ \bar{V}_{i,c} \end{bmatrix} \text{ remains constant} \tag{15}$$

A bus with any number of loads of any type, each connected in Y_g or Δ, any number of PQ generators and no Power-Voltage generators, is a PQ bus.

A bus with any number of loads of any type, each connected in Y_g or Δ, any number of PQ generators and at least one Power-Voltage generator, is a PV bus.

All types of loads are modelled in both the PFA and the SEA. The user chooses which model they prefer for each application. For example, the user may desire to model solar panels as a constant power (PQ) load instead of a constant current (I) load. The use of a variety of load types has transversal application and can be utilized on all three data availability levels to give more options to the user.

3.3.1. PFA MODELLING

PQ generators generate constant complex power and are described by:

$$\mathbf{\bar{S}_{G,i}} = \begin{bmatrix} \bar{S}_{G,i,a} \\ \bar{S}_{G,i,b} \\ \bar{S}_{G,i,c} \end{bmatrix} = \begin{bmatrix} \bar{V}_{i,a} * \mathrm{conj}(\bar{I}_{G,i,a}) \\ \bar{V}_{i,b} * \mathrm{conj}(\bar{I}_{G,i,b}) \\ \bar{V}_{i,c} * \mathrm{conj}(\bar{I}_{G,i,c}) \end{bmatrix} \text{remains constant} \quad (16)$$

Power-Voltage generators have constant voltage magnitude, generate constant real power and are described by:

$$\mathrm{abs}(\mathbf{\bar{V}_i}) = \begin{bmatrix} \mathrm{abs}(\bar{V}_{i,a}) \\ \mathrm{abs}(\bar{V}_{i,b}) \\ \mathrm{abs}(\bar{V}_{i,c}) \end{bmatrix} \text{ remains constant} \quad (17)$$

$$\mathrm{re}(\mathbf{\bar{S}_{G,i}}) = \begin{bmatrix} \mathrm{re}(\bar{S}_{G,i,a}) \\ \mathrm{re}(\bar{S}_{G,i,b}) \\ \mathrm{re}(\bar{S}_{G,i,c}) \end{bmatrix} \text{ remains constant} \quad (18)$$

If a PV bus has more than one Power-Voltage generator, all must have the same constant voltage.

For all loads, their complex power consumption is given by:

$$\mathbf{\bar{S}_{L,i}} = \begin{bmatrix} \bar{S}_{L,i,a} \\ \bar{S}_{L,i,b} \\ \bar{S}_{L,i,c} \end{bmatrix} = \begin{bmatrix} \bar{V}_{i,a} * \mathrm{conj}(\bar{I}_{L,i,a}) \\ \bar{V}_{i,b} * \mathrm{conj}(\bar{I}_{L,i,b}) \\ \bar{V}_{i,c} * \mathrm{conj}(\bar{I}_{L,i,c}) \end{bmatrix} \quad (19)$$

and their voltage drop is given by Ohm's Law:

$$\mathbf{\bar{V}_{L,i}} = \begin{bmatrix} \bar{V}_{L,i,a} \\ \bar{V}_{L,i,b} \\ \bar{V}_{L,i,c} \end{bmatrix} = \begin{bmatrix} \bar{Z}_{L,i,a} * \bar{I}_{L,i,a} \\ \bar{Z}_{L,i,b} * \bar{I}_{L,i,b} \\ \bar{Z}_{L,i,c} * \bar{I}_{L,i,c} \end{bmatrix} \quad (20)$$

PQ loads consume constant complex power and are described by:

$$\mathbf{\bar{S}_{L,i}} = \begin{bmatrix} \bar{S}_{L,i,a} \\ \bar{S}_{L,i,b} \\ \bar{S}_{L,i,c} \end{bmatrix} = \begin{bmatrix} \bar{V}_{i,a} * \mathrm{conj}(\bar{I}_{L,i,a}) \\ \bar{V}_{i,b} * \mathrm{conj}(\bar{I}_{L,i,b}) \\ \bar{V}_{i,c} * \mathrm{conj}(\bar{I}_{L,i,c}) \end{bmatrix} \text{remains constant} \quad (21)$$

I loads have steady current magnitude and constant power factor and are described by:

$$\mathrm{abs}(\mathbf{\bar{I}_{L,i}}) = \begin{bmatrix} \mathrm{abs}(\bar{I}_{L,i,a}) \\ \mathrm{abs}(\bar{I}_{L,i,b}) \\ \mathrm{abs}(\bar{I}_{L,i,c}) \end{bmatrix} \text{ remains constant} \quad (22)$$

$$\mathrm{arg}(\mathbf{\bar{S}_{L,i}}) = \begin{bmatrix} \mathrm{arg}(\bar{S}_{L,i,a}) \\ \mathrm{arg}(\bar{S}_{L,i,b}) \\ \mathrm{arg}(\bar{S}_{L,i,c}) \end{bmatrix} \text{ remains constant} \quad (23)$$

Z Loads have constant impedance and are described by:

$$\mathbf{\bar{Z}_{L,i}} = \begin{bmatrix} \bar{Z}_{i,a} \\ \bar{Z}_{i,b} \\ \bar{Z}_{i,c} \end{bmatrix} \text{ remains constant} \quad (24)$$

(21)-(24) refer to Y_g-connected loads. In the case of Δ-connected loads, the subscripts a, b, c are replaced by the subscripts ab, bc, ca respectively.

3.3.2. SEA MODELLING

The SEA Modelling of generators, loads and buses is the same as in the PFA Modelling.

3.4. TRANSFORMERS

One of the main advantages of AC current is the easy modification of voltage levels through transformers. Because there are three phases, three-phase transformers are used in modern power systems. Each three-phase transformer consists of three one-phase two-winding transformers, where the two windings have a given number of turns in the primary and secondary sides. The primary winding (P) and secondary winding (S) of each three-phase transformer can be connected in Y_g, Y or Δ. Of course, transformers are not ideal and have losses, which are modelled as an impedance equal to:

$$\bar{\mathbf{Z}} = \bar{Z} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \bar{Z} * \mathbf{Id} = (g + j * b) * \mathbf{Id}$$

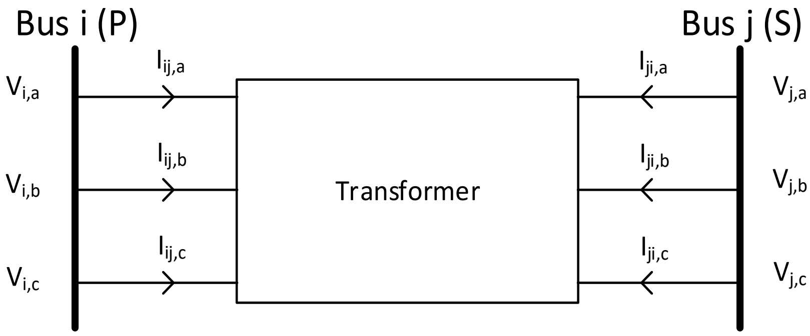

Figure 4: A transformer connected between bus i and bus j

Transformers are usually equipped with taps. If the taps are on the primary side, then by changing the tap, the primary winding turns can be changed. If the taps are on the secondary side, then by changing the tap, the secondary winding turns can be changed. For example, if a transformer has 20 taps on the primary side and can increase or decrease the primary winding turns by 10 % then each tap causes a 1 % increase or decrease.

3.4.1. PFA MODELLING

Depending on the transformer connection, different equations are used to relate the primary and secondary voltages with the primary and secondary currents. Primary current $\bar{\mathbf{I}}_\mathbf{P}$ is defined as $\bar{\mathbf{I}}_{\mathbf{ij}}$ and secondary current $\bar{\mathbf{I}}_\mathbf{s}$ is defined as $\bar{\mathbf{I}}_{\mathbf{ji}}$.These equations are [6]:

for Y_g-Y_g, Y_g-Δ, Y-Δ, Δ-Δ connections:

$$\bar{\mathbf{V}}_\mathbf{P}^{\alpha\beta0} = \mathbf{N_1} * \bar{\mathbf{V}}_\mathbf{S}^{\alpha\beta0} + \bar{Z} * \mathbf{N_2} * \bar{\mathbf{I}}_\mathbf{P}^{\alpha\beta0} \quad (25)$$

$$\mathbf{N_3} * \bar{\mathbf{I}}_\mathbf{P}^{\alpha\beta0} = \mathbf{N_4} * \bar{\mathbf{I}}_\mathbf{S}^{\alpha\beta0} \quad (26)$$

for Δ-Y_g connection:

$$\bar{\mathbf{V}}_{\mathbf{P}}^{\alpha\beta0} = \mathbf{N_1} * \bar{\mathbf{V}}_{\mathbf{S}}^{\alpha\beta0} + \bar{Z} * \mathbf{N_2} * \bar{\mathbf{I}}_{\mathbf{S}}^{\alpha\beta0} \qquad (27)$$

$$\mathbf{N_3} * \bar{\mathbf{I}}_{\mathbf{P}}^{\alpha\beta0} = \mathbf{N_4} * \bar{\mathbf{I}}_{\mathbf{S}}^{\alpha\beta0} \qquad (28)$$

The matrices involved are:

Primary	Secondary	N_1	N_2	N_3	N_4
Y_g	Y_g	$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$	$\begin{bmatrix}\mathbf{G}(-\varphi) & \vdots\\ \cdots & \delta(\varphi)\end{bmatrix}$	$-\begin{bmatrix}\mathbf{G}(\varphi) & \vdots\\ \cdots & \delta(\varphi)\end{bmatrix}$	**Id**
Y_g	Δ	$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$	$\begin{bmatrix}\mathbf{G}(-\varphi-30°) & \vdots\\ \cdots & \delta(\varphi+30°)\end{bmatrix}$	$-\begin{bmatrix}\mathbf{G}(\varphi) & \vdots\\ \cdots & 0\end{bmatrix}$	**Id**
Y	Δ	$\sqrt{3} * \begin{bmatrix}\mathbf{G}(30°) & \vdots\\ \cdots & 0\end{bmatrix}$	$\begin{bmatrix}\mathbf{G}(-\varphi) & \vdots\\ \cdots & 0\end{bmatrix}$	$-\begin{bmatrix}\mathbf{G}(\varphi) & \vdots\\ \cdots & 0\end{bmatrix}$	**Id**
Δ	Y_g	$\sqrt{3} * \begin{bmatrix}\mathbf{G}(30°-\varphi) & \vdots\\ \cdots & \delta(\varphi-30°)\end{bmatrix}$	$-\begin{bmatrix}\mathbf{G}(30°-\varphi) & \vdots\\ \cdots & \delta(\varphi-30°)\end{bmatrix}$	**Id**	$-\begin{bmatrix}\mathbf{G}(-\varphi) & \vdots\\ \cdots & 0\end{bmatrix}$
Δ	Δ	$\begin{bmatrix}\mathbf{G}(30°) & \vdots\\ \cdots & 0\end{bmatrix}$	$\begin{bmatrix}\mathbf{G}(-\varphi) & \vdots\\ \cdots & \delta(\varphi)\end{bmatrix}$	$-\begin{bmatrix}\mathbf{G}(\varphi) & \vdots\\ \cdots & \delta(\varphi)\end{bmatrix}$	**Id**

Table 1: Transformer matrices for the PSA

where

$$\mathbf{G}(\gamma) = \begin{bmatrix}\cos\gamma & -\sin\gamma\\ \sin\gamma & \cos\gamma\end{bmatrix}$$

$$\delta(\gamma) = \begin{cases}1 \text{ if } \dfrac{\gamma}{120} \in \text{integers}\\ -1 \text{ otherwise}\end{cases}$$

3.4.2. SEA MODELLING

In the SEA, transformers are treated as six port networks, whose currents relate to their voltages through [11]:

$$\begin{bmatrix}\bar{I}_{ij,a}\\ \bar{I}_{ij,b}\\ \bar{I}_{ij,c}\\ \bar{I}_{ji,a}\\ \bar{I}_{ji,b}\\ \bar{I}_{ji,c}\end{bmatrix} = \bar{\mathbf{Y}} * \begin{bmatrix}\bar{V}_{i,a}\\ \bar{V}_{i,b}\\ \bar{V}_{i,c}\\ \bar{V}_{j,a}\\ \bar{V}_{j,b}\\ \bar{V}_{j,c}\end{bmatrix} \qquad (29)$$

$$\bar{\mathbf{Y}} = \mathbf{G} + j * \mathbf{B} = \begin{bmatrix}\dfrac{\bar{\mathbf{A}}}{a^2} & \dfrac{\bar{\mathbf{C}}}{a*b}\\ \dfrac{\bar{\mathbf{C}}^{\mathbf{T}}}{a*b} & \dfrac{\bar{\mathbf{B}}}{b^2}\end{bmatrix} \qquad (30)$$

where

a is the primary tap factor and is equal to 1 if the nominal tap is used for the primary winding, and b is the secondary tap factor and is equal to 1 if the nominal tap is used for the secondary winding.

The matrices involved are [4]:

Type	Primary	Secondary	$\bar{A}$	$\bar{B}$	$\bar{C}$
Step-Down	Y_g	Y_g	$\bar{\mathbf{I}}$	$\bar{\mathbf{I}}$	$-\bar{\mathbf{I}}$
Step-Down	Y_g	Δ	$\bar{\mathbf{I}}$	$\bar{\mathbf{II}}$	$\bar{\mathbf{III}}$
Step-Down	Δ	Y_g	$\bar{\mathbf{II}}$	$\bar{\mathbf{I}}$	$\bar{\mathbf{III}}$
Step-Down	Δ	Y	$\bar{\mathbf{II}}$	$\bar{\mathbf{II}}$	$\bar{\mathbf{III}}$
Step-Down	Δ	Δ	$\bar{\mathbf{II}}$	$\bar{\mathbf{II}}$	$-\bar{\mathbf{II}}$
Step-Up	Y_g	Y_g	$\bar{\mathbf{I}}$	$\bar{\mathbf{I}}$	$-\bar{\mathbf{I}}$
Step-Up	Y_g	Δ	$\bar{\mathbf{I}}$	$\bar{\mathbf{II}}$	$\bar{\mathbf{III}}^T$
Step-Up	Δ	Y_g	$\bar{\mathbf{II}}$	$\bar{\mathbf{I}}$	$\bar{\mathbf{III}}^T$
Step-Up	Δ	Y	$\bar{\mathbf{II}}$	$\bar{\mathbf{II}}$	$\bar{\mathbf{III}}^T$
Step-Up	Δ	Δ	$\bar{\mathbf{II}}$	$\bar{\mathbf{II}}$	$-\bar{\mathbf{II}}$

Table 2: Transformer matrices for the SEA

where

$$\bar{\mathbf{I}} = \begin{bmatrix} g & 0 & 0 \\ 0 & g & 0 \\ 0 & 0 & g \end{bmatrix} + j * \begin{bmatrix} b & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{bmatrix}$$

$$\bar{\mathbf{II}} = \frac{1}{3} * \begin{bmatrix} 2*g & -g & -g \\ -g & 2*g & -g \\ -g & -g & 2*g \end{bmatrix} + j * \frac{1}{3} * \begin{bmatrix} 2*b & -b & -b \\ -b & 2*b & -b \\ -b & -b & 2*b \end{bmatrix}$$

$$\bar{\mathbf{III}} = \frac{1}{\sqrt{3}} * \begin{bmatrix} -g & g & 0 \\ 0 & -g & g \\ g & 0 & -g \end{bmatrix} + j * \frac{1}{\sqrt{3}} * \begin{bmatrix} -b & b & 0 \\ 0 & -b & b \\ b & 0 & -b \end{bmatrix}$$

3.5. DISTRIBUTION LINES

Electric power is distributed to the loads through distribution lines. Distribution lines are made up of three phase conductors a, b, c and in some cases one neutral conductor. Aside from the voltage drop of each conductor due to its own current, each conductor also has a voltage drop associated with the current of all other conductors present.

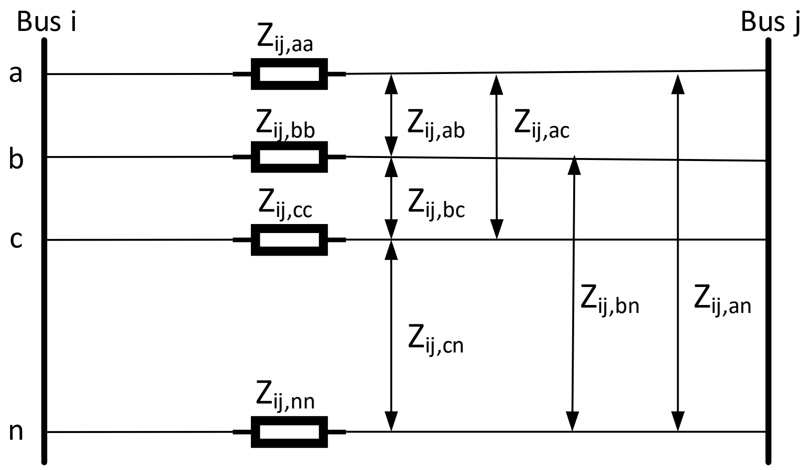

Figure 5: A distribution line connected between bus i and bus j with its impedances visible

If there is a neutral conductor, the voltage drop on each conductor connected between bus i and bus j is given by [12]:

$$\begin{bmatrix}\bar{V}_{i,a}\\ \bar{V}_{i,b}\\ \bar{V}_{i,c}\\ \bar{V}_{i,n}\end{bmatrix} - \begin{bmatrix}\bar{V}_{j,a}\\ \bar{V}_{j,b}\\ \bar{V}_{j,c}\\ \bar{V}_{j,n}\end{bmatrix} = \bar{\mathbf{Z}}_{\mathbf{abcn}} * \begin{bmatrix}\bar{I}_{ij,a}\\ \bar{I}_{ij,b}\\ \bar{I}_{ij,c}\\ \bar{I}_{ij,n}\end{bmatrix} \tag{31}$$

where

$$\bar{\mathbf{Z}}_{\mathbf{abcn}} = \begin{bmatrix}\bar{\mathbf{Z}}_{\mathbf{1}} & \bar{\mathbf{Z}}_{\mathbf{2}}\\ \bar{\mathbf{Z}}_{\mathbf{3}} & \bar{\mathbf{Z}}_{\mathbf{4}}\end{bmatrix} \tag{32}$$

$$\bar{\mathbf{Z}}_{\mathbf{1}} = \begin{bmatrix}\bar{z}_{aa} & \bar{z}_{ab} & \bar{z}_{ac}\\ \bar{z}_{ba} & \bar{z}_{bb} & \bar{z}_{bc}\\ \bar{z}_{ca} & \bar{z}_{cb} & \bar{z}_{cc}\end{bmatrix} \tag{33}$$

$$\bar{\mathbf{Z}}_{\mathbf{2}} = \begin{bmatrix}\bar{z}_{an}\\ \bar{z}_{bn}\\ \bar{z}_{cn}\end{bmatrix} \tag{34}$$

$$\bar{\mathbf{Z}}_{\mathbf{3}} = \begin{bmatrix}\bar{z}_{na} & \bar{z}_{nb} & \bar{z}_{nc}\end{bmatrix} \tag{35}$$

$$\bar{\mathbf{Z}}_{\mathbf{4}} = \begin{bmatrix}\bar{z}_{nn}\end{bmatrix} \tag{36}$$

The elements of $\bar{\mathbf{Z}}_{\mathbf{abcn}}$ are in Ω/mile and given by :

$$\bar{z}_{kk} = r_k + 0.00158836 * f + j * 0.00202237 * f * (\ln\frac{1}{GMR_k} + 7.6786 + 0.5 * \ln\frac{100}{f}) \tag{37}$$

$$\bar{z}_{kl} = 0.00158836 * f + j * 0.00202237 * f * (\ln\frac{1}{D_{kl}} + 7.6786 + 0.5 * \ln\frac{100}{f}) \tag{38}$$

$k, l \in \{a, b, c, n\}$

r_k is the resistance of conductor k in Ω/mile

f is the frequency of the power system in Hz

R_k the radius of conductor k in ft

$GMR_k = R_k * e^{-\frac{1}{4}}$ is the geometric mean radius of conductor k in ft

D_{kl} is the distance between conductor k and conductor l in ft

Since only phases a, b, c are of interest the 4 x 4 matrix $\bar{\mathbf{Z}}_{\mathbf{abcn}}$ can be reduced to a 3 x 3 matrix $\bar{\mathbf{Z}}_{\mathbf{ij}}$ by utilizing Kron reduction [12]:

$$\bar{\mathbf{Z}}_{\mathbf{ij}} = \begin{bmatrix} \bar{Z}_{ij,aa} & \bar{Z}_{ij,ab} & \bar{Z}_{ij,ac} \\ \bar{Z}_{ij,ba} & \bar{Z}_{ij,bb} & \bar{Z}_{ij,bc} \\ \bar{Z}_{ij,ca} & \bar{Z}_{ij,cb} & \bar{Z}_{ij,cc} \end{bmatrix} = \bar{\mathbf{Z}}_{\mathbf{abc}} = \bar{\mathbf{Z}}_{\mathbf{1}} - \bar{\mathbf{Z}}_{\mathbf{2}} * \bar{\mathbf{Z}}_{\mathbf{4}}^{-1} * \bar{\mathbf{Z}}_{\mathbf{3}} \tag{39}$$

If there is no neutral conductor, then $\bar{\mathbf{Z}}_{\mathbf{ij}} = \bar{\mathbf{Z}}_{\mathbf{abcn}}$.

After determining $\bar{\mathbf{Z}}_{\mathbf{ij}}$, the voltage drop of each phase can be determined as a function of the current of each phase:

$$\begin{bmatrix} \bar{V}_{i,a} \\ \bar{V}_{i,b} \\ \bar{V}_{i,c} \end{bmatrix} - \begin{bmatrix} \bar{V}_{j,a} \\ \bar{V}_{j,b} \\ \bar{V}_{j,c} \end{bmatrix} = \bar{\mathbf{Z}}_{\mathbf{ij}} * \begin{bmatrix} \bar{I}_{ij,a} \\ \bar{I}_{ij,b} \\ \bar{I}_{ij,c} \end{bmatrix} \tag{40}$$

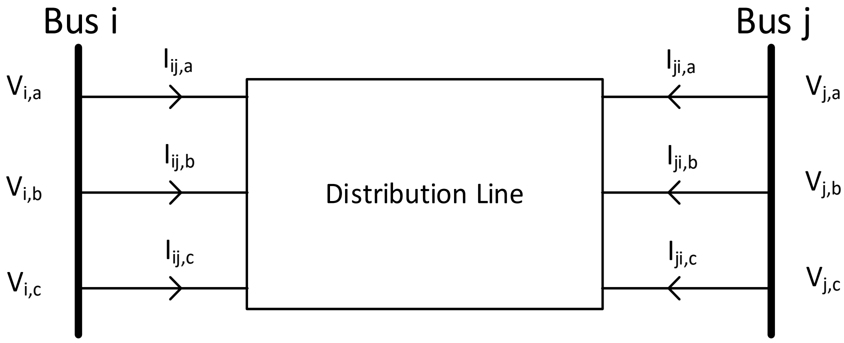

Figure 6: A distribution line connected between bus i and bus j

3.5.1. PFA MODELLING

Converting (40) into the αβ0 system:

$$\bar{\mathbf{V}}_{\mathbf{i}}^{\boldsymbol{\alpha\beta 0}} - \bar{\mathbf{V}}_{\mathbf{j}}^{\boldsymbol{\alpha\beta 0}} = \mathbf{A}^{-1} * \bar{\mathbf{Z}}_{\mathbf{ij}} * \mathbf{A} * \bar{\mathbf{I}}_{\mathbf{ij}}^{\boldsymbol{\alpha\beta 0}} \tag{41}$$

or equivalently:

$$\bar{\mathbf{V}}_{\mathbf{i}}^{\boldsymbol{\alpha\beta 0}} = \mathbf{N_1} * \bar{\mathbf{V}}_{\mathbf{j}}^{\boldsymbol{\alpha\beta 0}} + \mathbf{N_2} * \bar{\mathbf{I}}_{\mathbf{ij}}^{\boldsymbol{\alpha\beta 0}} \tag{42}$$

where

$$\mathbf{N_1} = \mathbf{Id}$$
$$\mathbf{N_2} = \mathbf{A}^{-1} * \bar{\mathbf{Z}}_{\mathbf{ij}} * \mathbf{A}$$

and

$$\mathbf{N_3} * \bar{\mathbf{I}}_{\mathbf{ij}}^{\boldsymbol{\alpha\beta 0}} = \mathbf{N_4} * \bar{\mathbf{I}}_{\mathbf{ji}}^{\boldsymbol{\alpha\beta 0}} \tag{43}$$
$$\mathbf{N_3} = \mathbf{Id}$$
$$\mathbf{N_4} = -\mathbf{Id}$$

From (41), it is evident that the impedance of the line in the αβ0 system is defined as:

$$\mathbf{Z}_{\mathbf{ij}}^{\boldsymbol{\alpha\beta 0}} = \mathbf{A}^{-1} * \bar{\mathbf{Z}}_{\mathbf{ij}} * \mathbf{A} \tag{44}$$

Additional impedances between phases and between phases and ground can be easily included as Z loads.

3.5.2. SEA MODELLING

In the SEA, distribution lines are treated as six port networks, whose currents relate to their voltages through:

$$\begin{bmatrix} \overline{I}_{ij,a} \\ \overline{I}_{ij,b} \\ \overline{I}_{ij,c} \\ \overline{I}_{ji,a} \\ \overline{I}_{ji,b} \\ \overline{I}_{ji,c} \end{bmatrix} = \overline{\mathbf{Y}} * \begin{bmatrix} \overline{V}_{i,a} \\ \overline{V}_{i,b} \\ \overline{V}_{i,c} \\ \overline{V}_{j,a} \\ \overline{V}_{j,b} \\ \overline{V}_{j,c} \end{bmatrix} \quad (45)$$

$$\overline{\mathbf{Y}} = \mathbf{G} + \mathrm{j} * \mathbf{B} = \begin{bmatrix} \overline{\mathbf{Z}}_{\mathbf{ij}}{}^{-1} & -\overline{\mathbf{Z}}_{\mathbf{ij}}{}^{-1} \\ -\overline{\mathbf{Z}}_{\mathbf{ij}}{}^{-1} & \overline{\mathbf{Z}}_{\mathbf{ij}}{}^{-1} \end{bmatrix} \quad (46)$$

Additional impedances between phases and between phases and ground can be easily included as Z loads.

4. HIGH DATA AVAILABILITY SCENARIO

4.1. SCENARIO FOR LINE INFORMATION

The grid topology is complete, and just deviations or mistakes related with the conductor type may be checked.

The data availability scenario regarding line information is summarised in the table below:

Parameters for Lines	
Coordinates of line terminals (Xi,Yi/Xj,Yj)	YES (minor mistakes)
Length (km)	YES
Rated Voltage (kV)	YES
Rated Current (kA)	YES
Cable / OHL	YES
3-phase or 1-phase line	YES
If 1-phase line, phase	YES
Conductor Type	YES (mistakes possible)
AC-Resistance R' (20°C) (Ohm/km)	YES
Reactance X (Ohm/km)	YES
Max. Operational Temperature (°C)	YES
Conductor Material	YES
Susceptance (µS/km)	YES
Capacitance (µF/km)	YES

Table 3: High data availability scenario for lines

4.2. SCENARIO FOR LOAD INFORMATION

Consumption per phase of 3-phase customers is still unknown. High quality of data may enable estimation of imbalances at 3-phase customers. Historical or real-time data still contains errors, but less frequently.

The data availability scenario regarding load information is summarised in the table below:

Parameters for Loads	High
Power contracted	YES
Phase	YES
Latitude/Longitude	YES
Cost of load shedding	NO
Smart metering	YES
Smart meter measurements	
Active power P	YES
Reactive power Q	YES
Voltage V	YES
Current I	YES
Smart meters at all customers	YES
Hourly load curves	YES
Length of historical time series	Min. 1 year

Table 4: High data availability scenario for loads

4.3. SCENARIO FOR TRANSFORMER INFORMATION

All parameters are known and available, but tap positions are not optimized. Historical data series may still have some errors.

The data availability scenario regarding transformer information is summarised in the table below:

Parameters for Transformers	High
Rated Power (MVA)	YES
Rated Voltage MV-Side (kV)	YES
Rated Voltage LV-Side (kV)	YES
Vector group MV-Side	YES
Vector group LV-Side	YES
Phase shift (*30 deg)	YES
Positive Sequence Impedance Reactance X1 (p.u.)	YES
Positive Sequence Impedance Resistance R1 (p.u.)	YES
No load current (%)	YES
No load losses (kW)	YES
Tap changer (discrete/continuous)	YES
Tap position	YES
Tap position optimization	NO
Measurements	
Aggregated LV measurements	YES
Measurements per Feeder	YES
Length of historical time series	Min. 1 year

Table 5: High data availability scenario for transformers

4.4. SCENARIOS FOR SOLAR GENERATOR INFORMATION

Hourly load curves are available but must be checked for corrections. Historical data is also available, but it must be checked for possible minor deviations.

The data availability scenario regarding solar information is summarised in the table below.

Parameters for solar generators	High
Nominal Apparent Power (MVA)	YES
Operating point	
Local controller (Const. Q/ Const. V/ Q(P) Characteristic/ Const. Cosine etc.)	YES
Active power (kW)	NO
Power Factor	NO
Measurements	
Hourly load curves	YES
Disaggregated measurements of generation (at prosumers)	YES
Length of historical time series	Min. 1 year

Table 6: High data availability scenario for solar generators

5. METHODS

5.1. POWER FLOW ALGORITHM (PFA)

5.1.1. INTRODUCTION

The objective of this algorithm is to calculate the state variables of the power system given the states of its generators and loads. In general, this information is not always available and does not take into account errors in the data given. These faults make the SEA a better option to calculate the network's states. However, data may not be sufficient to run SEA. In that case, the PFA can be used to calculate certain power flows or injections of the power system that will be used in conjunction with the available data to use the SEA. The PFA models all the available components, creates the equations for each branch and solves the resulting system of equations to calculate the state variables.

5.1.2. INPUT AND OUTPUT

Input:

- the topology of the power system
- the rated voltages, rated power, type and necessary constants of every generator and load (loads can be from either measurements or load profiles, UC 3.19)
- the rated power, length and impedances per unit of length for each distribution line
- the rated voltages, rated power, configuration, impedance, transformer ratio and tap details and tap settings of every transformer
- the rated voltages, rated power and regulator ratio of every regulator
- the magnitudes and angles of the three phases of the slack bus

Output:

- the state variables vector x (UC 3.01), which, for the PFA, is defined as:

$$\mathbf{x}_{\textbf{Power Flow}} = \begin{bmatrix} V_{2,a} \\ V_{2,b} \\ V_{2,c} \\ \vdots \\ V_{n,a} \\ V_{n,b} \\ V_{n,c} \\ \delta_{2,a} \\ \delta_{2,b} \\ \delta_{2,c} \\ \vdots \\ \delta_{n,a} \\ \delta_{n,b} \\ \delta_{n,c} \end{bmatrix}$$

where n is the number of buses of the power system.

5.1.3. PSEUDOCODE

Initialization:
for every line ij in Network:
  calculate $\mathbf{N_1}$, $\mathbf{N_2}$, $\mathbf{N_3}$, $\mathbf{N_4}$ based on the component existing on the branch and convert them to 6 x 6 matrices
for every bus i in Network:
  initialize voltage $\overline{\mathbf{V}}_{\mathbf{i}} = \begin{bmatrix} \overline{V}_{i,a} \\ \overline{V}_{i,b} \\ \overline{V}_{i,c} \end{bmatrix} = \begin{bmatrix} e^{j*0°} \\ e^{-j*120°} \\ e^{j*120°} \end{bmatrix}$ pu, convert it to the αβ0 system using $\overline{\mathbf{V}}_{\mathbf{i}}^{\boldsymbol{\alpha\beta0}} = \mathbf{A^T} * \overline{\mathbf{V}}_{\mathbf{i}}$ and subsequently to a 6 x 1 vector

Backward Sweep:
for every level lvl of the Network starting from the bottom:
  for every line ij in lvl:
    calculate the 3 x 1 currents vector, $\mathbf{I_L}$, flowing through the loads of node **j** using calculations in abc and not in pu, convert it to the αβ0 system and then to a 6 x 1 vector
    calculate the 3 x 1 currents vector, $\mathbf{I_G}$, produced by the generators of node **j** using calculations in abc and not in pu, convert it to the αβ0 system and then to a 6 x 1 vector
    set $\overline{\mathbf{I}}^{\boldsymbol{\alpha\beta0}} = \overline{\mathbf{I}}_{\mathbf{L}}^{\boldsymbol{\alpha\beta0}} + \overline{\mathbf{I}}_{\mathbf{G}}^{\boldsymbol{\alpha\beta0}}$
    set $\overline{\mathbf{I}}_{\mathbf{D}}^{\boldsymbol{\alpha\beta0}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$
    for every descendant k of j: # Calculate the total current, $\mathbf{I_D}$, flowing through the lines connecting j to its descendants.
      calculate current of jk, $\mathbf{I_{jk}}$, in αβ0 and convert it into a 6 x 1 vector.
      using ij's $\mathbf{N_4}$, perform the following calculation: $\overline{\mathbf{I}}_{\mathbf{D}}^{\boldsymbol{\alpha\beta0}} = \overline{\mathbf{I}}_{\mathbf{D}}^{\boldsymbol{\alpha\beta0}} + \mathbf{N_4} * \overline{\mathbf{I}}_{\mathbf{jk}}^{\boldsymbol{\alpha\beta0}}$
    set $\overline{\mathbf{I}}^{\boldsymbol{\alpha\beta0}} = \overline{\mathbf{I}}^{\boldsymbol{\alpha\beta0}} - \overline{\mathbf{I}}_{\mathbf{D}}^{\boldsymbol{\alpha\beta0}}$
    using ij's $\mathbf{N_3}$, perform the following calculation: $\mathbf{I_{ij}} = \mathbf{N_3^{-1}} * \mathbf{I}$

Forward Sweep:
for every level lvl of the Network starting from the top:
  for every line ij in lvl:
  using ij's $\mathbf{N_1}$, $\mathbf{N_2}$ and i's voltages, $\mathbf{V_i}$, as reference, calculate j's voltages:

$$\overline{\mathbf{V}}_{\mathbf{j}}^{\boldsymbol{\alpha\beta0}} = \mathbf{N_2^{-1}} * (\overline{\mathbf{V}}_{\mathbf{i}}^{\boldsymbol{\alpha\beta0}} - \overline{\mathbf{Z}}_{\mathbf{ij}} * \mathbf{N_1} * \overline{\mathbf{V}}_{\mathbf{i}}^{\boldsymbol{\alpha\beta0}})$$

Error Calculation:
if the difference between the previous and the current state is lower than a certain threshold, ε, for every bus:
  return the voltages of all nodes
else
  return to **Backward Sweep**

5.1.4. FLOWCHART

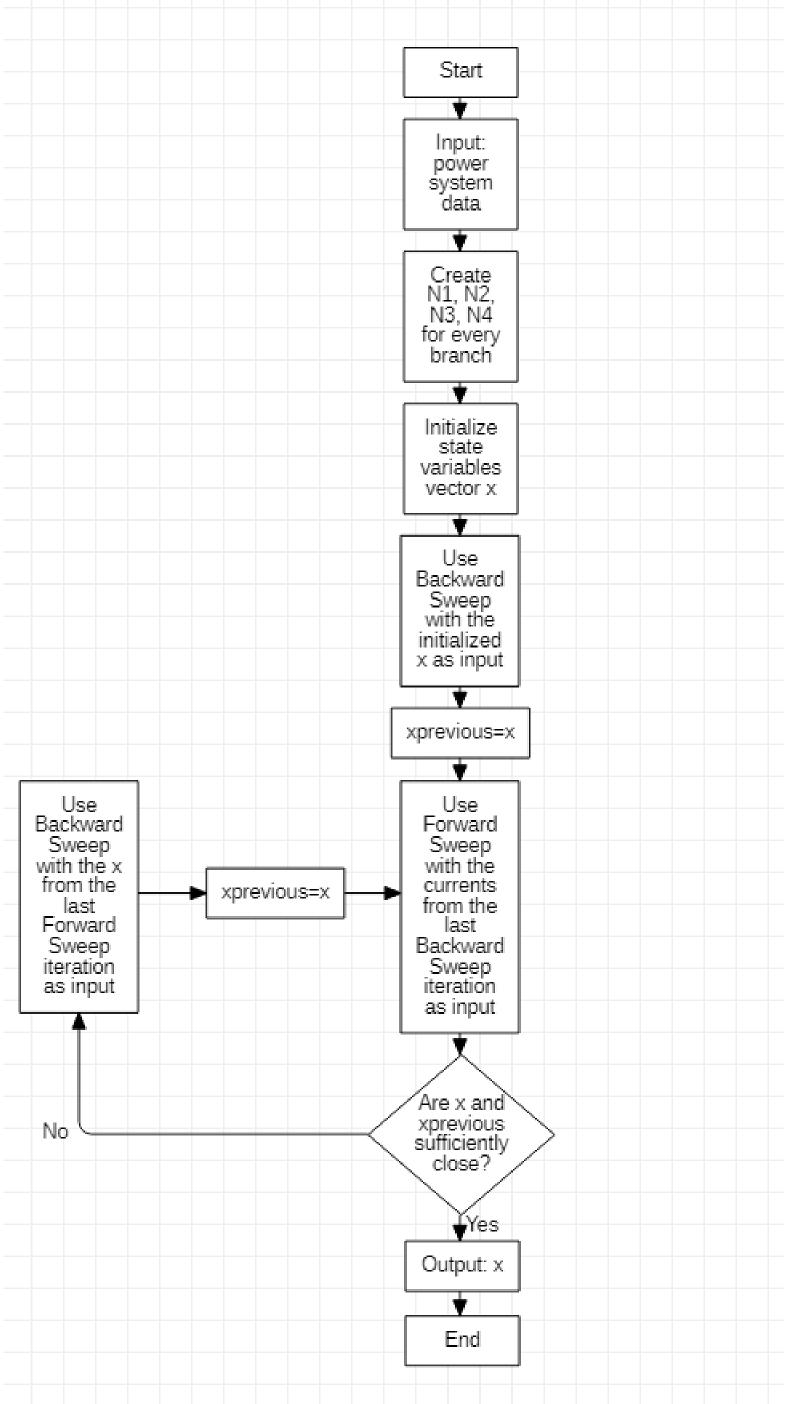

Figure 7: PFA Flowchart

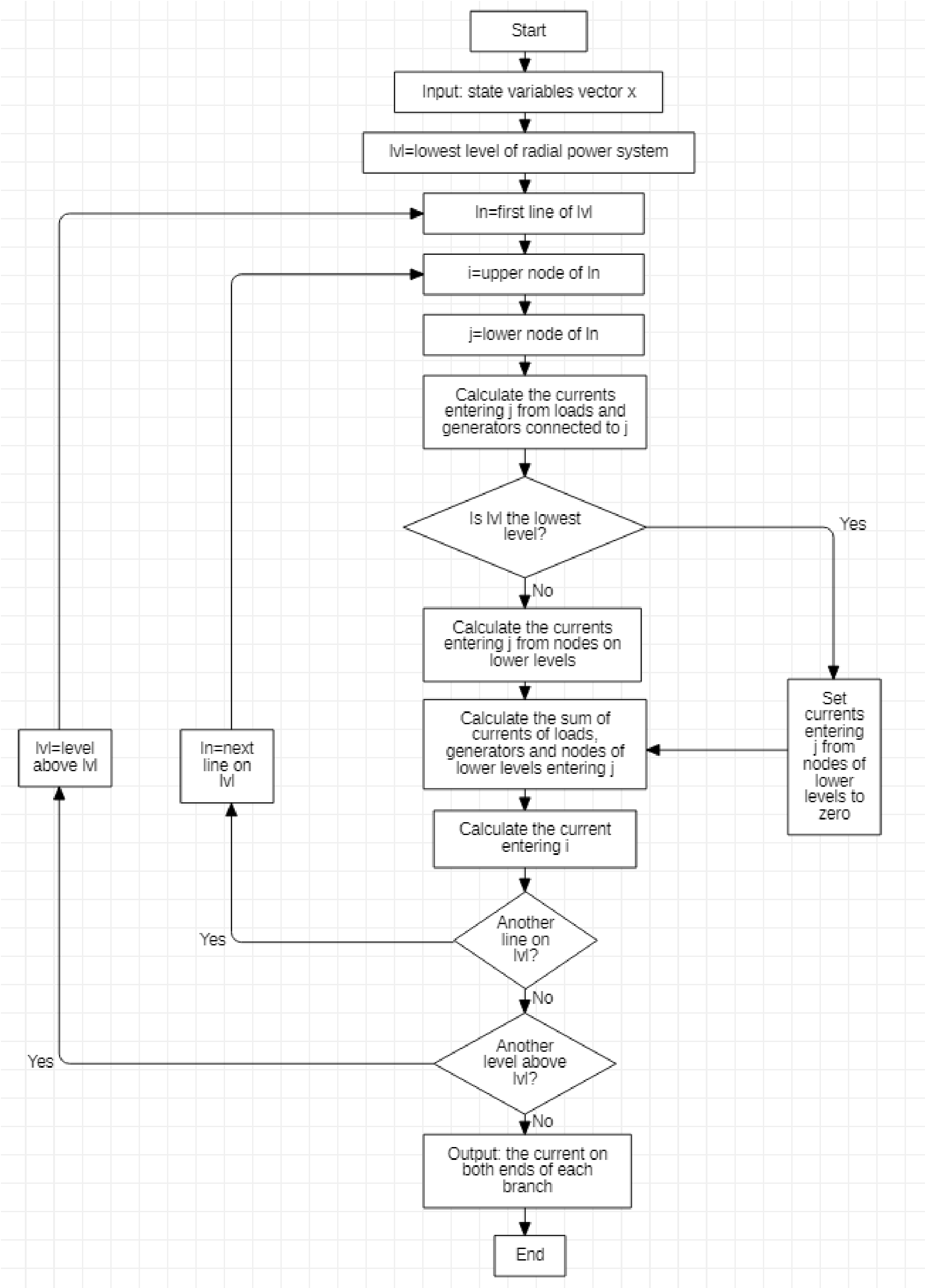

Figure 8: Backward Sweep Flowchart

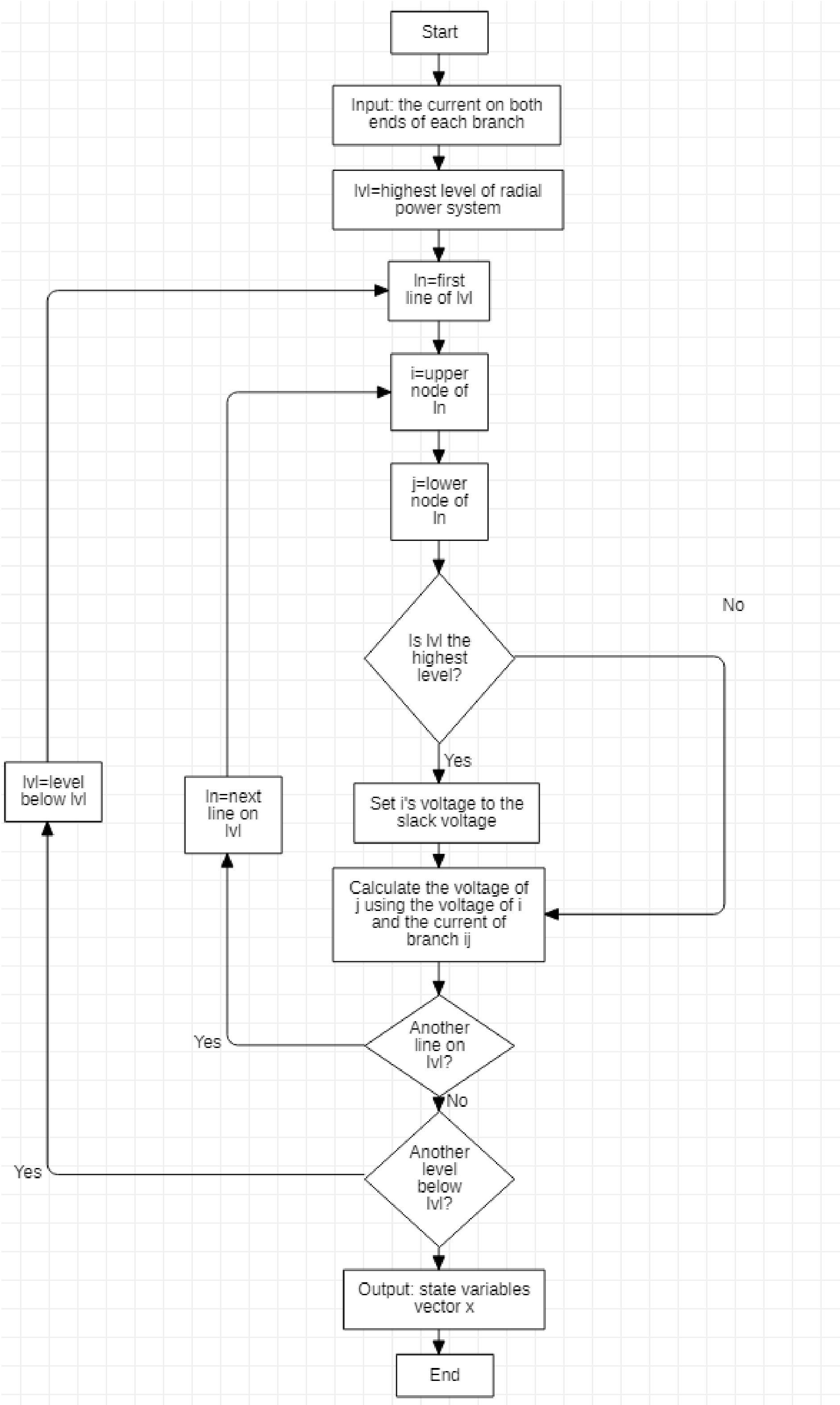

Figure 9: Forward Sweep Flowchart

5.2. TAP OPTIMISATION ALGORITHM (TOA)

5.2.1. INTRODUCTION

The objective of this function is to find the optimal tap settings of each transformer in the network. It uses a very straightforward brute force approach, utilizing an exhaustive search to find the globally optimal settings combination.

5.2.2. INPUT AND OUTPUT

Input:

- the topology of the power system
- the rated voltages, rated power, type and necessary constants of every generator and load (loads can be from either measurements or load profiles, UC 3.19)
- the rated power, length and impedances per unit of length for each distribution line
- the rated voltages, rated power, configuration, impedance, transformer ratio and tap details and tap settings of every transformer
- the rated voltages, rated power and regulator ratio of every regulator
- the magnitudes and angles of the three phases of the slack bus
- the weight vector w, which is defined as:

$$\mathbf{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$$

where n is the number of buses of the power system (the closer the user desires a bus to have a voltage close to its nominal voltage the higher its corresponding weight).

Output:

- the tap settings vector t, which is defined as:

$$\mathbf{t} = \begin{bmatrix} t_1 \\ \vdots \\ t_k \end{bmatrix}$$

where k is the number of transformers of the power system.

5.2.3. PSEUDOCODE

for every tap setting combination t of the k transformers

use power flow to calculate the voltage magnitudes of all buses

calculate

$$C(x) = \sum_{i=1}^{n} w_i * (V_i - 1)^2$$

if this is the first iteration of the loop set t_{min}=t and min=C(x)

if C(x)<t set t_{min}=t and min=C(x)

return t_{min}

5.2.4. FLOWCHART

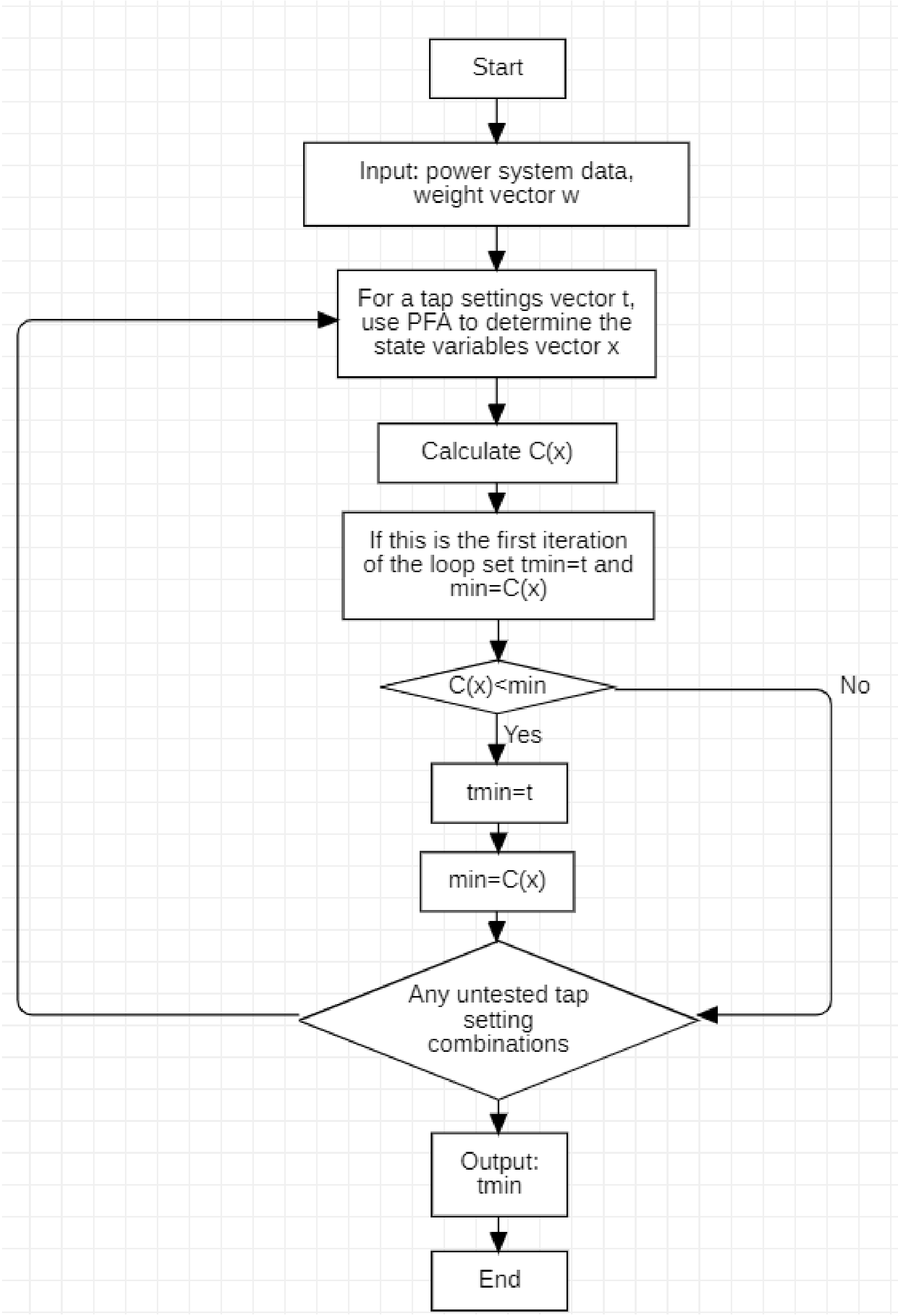

Figure 10: TAO Flowchart

5.3. STATE ESTIMATION ALGORITHM (SEA)

5.3.1. INTRODUCTION

The objective of this algorithm is to find the state variables of the power system, but it requires many measurements from the network in order to produce a result of satisfying accuracy. In case the measurements are not enough, results from the PFA supplement the sufficient results. The algorithm also takes into account the errors in the measurements by minimizing the sum of the square of errors of the measurements.

BAD DATA DETECTION

The SEA has a bad data detection algorithm integrated in it. This algorithm uses the chi-squared test to determine which measurements are faulty, due to e.g. cyber-attacks or technical malfunction.

5.3.2. INPUT AND OUTPUT

Input:

- the topology of the power system
- the rated power, length and impedances per unit of length for each distribution line
- the rated voltages, rated power, configuration, impedance, transformer ratio and tap details and tap settings (optional) of every transformer
- the rated voltages, rated power and regulator ratio of every regulator
- the angles of the three phases of the slack bus
- the power system measurements vector z along with the respective functions h (see Annex), which is defined as:

$$\mathbf{z} = \begin{bmatrix} z_1 \\ \vdots \\ z_m \end{bmatrix} = \begin{bmatrix} h_1(\mathbf{x}_{\mathbf{State\ Estimation}}) \\ \vdots \\ h_m(\mathbf{x}_{\mathbf{State\ Estimation}}) \end{bmatrix}$$

  where m is the number of available measurements of the power system
- the covariance matrix for the measurements, R, which is defined as:

$$\mathbf{R} = \begin{bmatrix} \sigma_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_m^2 \end{bmatrix} = \mathrm{diag}(\sigma_1^2, \dots, \sigma_m^2)$$

Output:

- the state variables vector x (UC 3.02), which, for the SEA, is defined as:

$$\mathbf{x}_{\textbf{State Estimation}} = \begin{bmatrix} V_{1,a} \\ V_{1,b} \\ V_{1,c} \\ \vdots \\ V_{n,a} \\ V_{n,b} \\ V_{n,c} \\ \delta_{2,a} \\ \delta_{2,b} \\ \delta_{2,c} \\ \vdots \\ \delta_{n,a} \\ \delta_{n,b} \\ \delta_{n,c} \end{bmatrix}$$

where n is the number of buses of the power system (the vector x can also contain the tap settings a and b of equation (36) of any number of transformers, thus addressing UC 3.21).

- the load power vector $\bar{\mathbf{S}}$ **(**UC 3.13, UC 3.31), which is defined as:

$$\bar{\mathbf{S}} = \begin{bmatrix} \bar{\mathbf{S}}_{\mathbf{L,1,a}} \\ \bar{\mathbf{S}}_{\mathbf{L,1,b}} \\ \bar{\mathbf{S}}_{\mathbf{L,1,c}} \\ \vdots \\ \bar{\mathbf{S}}_{\mathbf{L,n,a}} \\ \bar{\mathbf{S}}_{\mathbf{L,n,b}} \\ \bar{\mathbf{S}}_{\mathbf{L,n,c}} \end{bmatrix}$$

5.3.3. PSEUDOCODE

Initialization:
the input z contains m measurements and their covariance matrix

$$\mathbf{R} = \mathrm{diag}(\sigma_1^2, \ldots, \sigma_m^2)$$

```
# the output x contains s variables to be calculated
# m+2*b must be greater than s in order to use the SEA, where b is the number of buses with no PQ
  loads and generators
# greater values of m lead to more accurate results
if m+2*b<=s
  if there are enough measurements for the PFA
    use PFA to create pseudomeasurements
  else
    retun 'not enough measurements'
for every measurement zi
  create the corresponding function hi (see Annex)
```

Constraints:
for every bus with no PQ loads and no generators
create the zero real and zero reactive power injection constraints for that bus (see Annex)
create

$$\mathbf{H} = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \cdots & \frac{\partial h_1}{\partial x_s} \\ \vdots & \ddots & \vdots \\ \frac{\partial h_m}{\partial x_1} & \cdots & \frac{\partial h_m}{\partial x_s} \end{bmatrix}$$

the SEA is very sensitive on the variances of the measurements and convergence may not be achieved if they are not accurate
create

$$\mathbf{R^{-1}} = \mathrm{diag}(\frac{1}{\sigma_1^2}, \dots, \frac{1}{\sigma_m^2})$$

create

$$\mathbf{G} = \mathbf{H^T} * \mathbf{R^{-1}} * \mathbf{H}$$

find the **x** that minimizes

$$C(\mathbf{x}) = \sum_{i=1}^{m} \frac{1}{\sigma_i^2} * [z_i - h_i(\mathbf{x})]^2$$

subject to 2*b constraints found in **Constraints**

Bad data detection:
create

$$\mathbf{P} = \mathbf{R} - \mathbf{H}(\mathbf{x}) * \mathbf{G^{-1}}(\mathbf{x}) * \mathbf{H^T}(\mathbf{x})$$

calculate

$$C(\mathbf{x}) = \sum_{i=1}^{m} \frac{1}{\sigma_i^2} * [z_i - h_i(\mathbf{x})]^2$$

set

$$\mathrm{Limit} = \chi^2_{m-s,0,99}$$

where $\chi^2_{m-s,0,99}$ is defined as the value for which

$$0,99 = \int_0^{\chi_{m-s,0,99}} \frac{x^{\frac{m-s}{2}-1} * e^{-\frac{x}{2}}}{2^{\frac{m-s}{2}} * \Gamma\left(\frac{m-s}{2}\right)} d(m-s)$$

($\chi^2_{m-s,0,99}$ can be derived from the tables of the chi-square distribution)
if C(**x**)>Limit
for every measurement z_i
calculate

$$\text{Standarized error of measurement i} = \frac{z_i - h_i(\mathbf{x})}{\sqrt{P_{ii}}}$$

discard the measurement with the largest standardized error
return to **Initialization** with one less measurement as input
calculate $\bar{\mathbf{S}}$ (see Annex)
return **x**, $\bar{\mathbf{S}}$

5.3.4. FLOWCHART

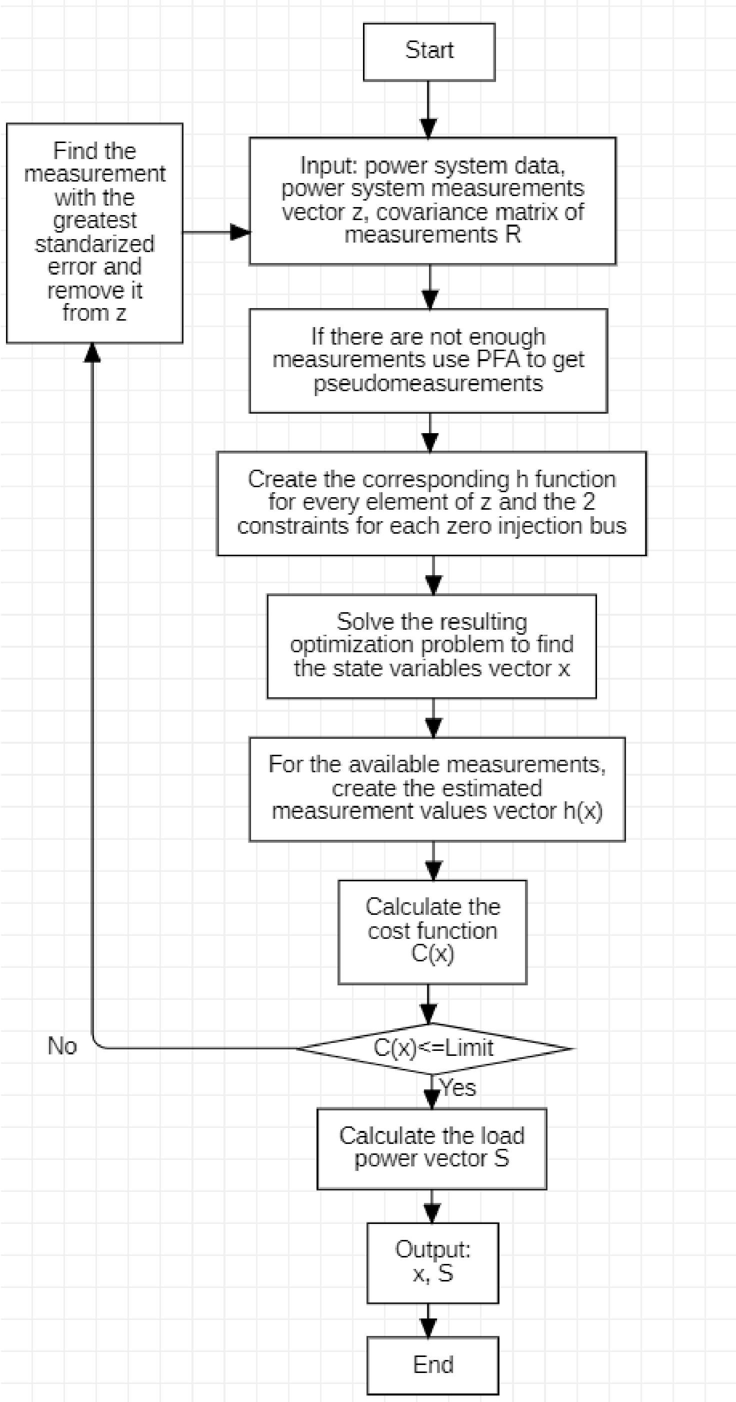

Figure 11: SEA Flowchart

6. VALIDATION

6.1. POWER FLOW ALGORITHM (PFA)

The PFA was tested on all the IEEE feeder test cases. Some indicative results are shown below.

6.1.1. IEEE 4-BUS FEEDER (STEP DOWN $Y_G D$)

Bus	V_a	δ_a	V_b	δ_b	V_c	δ_c
2	7111.103	-0.205	7143.654	-120.430	7111.180	119.537
3	3893.741	-2.824	3973.147	-123.860	3876.752	115.729
4	3422.745	-5.761	3647.783	-130.300	3299.480	108.620

6.1.2. IEEE 13-BUS FEEDER

Bus	V_a	δ_a	V_b	δ_b	V_c	δ_c
RG1	1.062	0.000	1.050	-120.000	1.069	120.000
632	1.021	-2.489	1.042	-121.720	1.018	117.829
645	1.021	-2.489	1.033	-121.900	1.016	117.856
633	1.018	-2.554	1.040	-121.770	1.015	117.825
671	0.990	-5.296	1.053	-122.340	0.978	116.026
646	1.021	-2.489	1.031	-121.980	1.013	117.902
634	0.994	-3.230	1.022	-122.220	0.996	117.346
684	0.988	-5.319	1.053	-122.340	0.976	115.925
680	0.990	-5.296	1.053	-122.340	0.978	116.026
692	0.990	-5.296	1.053	-122.340	0.978	116.026
652	0.983	-5.245	1.053	-122.340	0.976	115.925
611	0.988	-5.319	1.053	-122.340	0.974	115.779
675	0.984	-5.546	1.055	-122.520	0.976	116.041

6.1.3. IEEE EUROPEAN LOW VOLTAGE TEST FEEDER

The European LV Test Feeder contains 907 buses. The results were accurate. The maximum error observed was 1 Volt for the voltage magnitudes and 0.005 degrees for the voltage angles.

6.2. STATE ESTIMATION ALGORITHM (SEA)

For the state estimation algorithm, there were no test cases available that could be directly utilized. The PFA was used to create the desired inputs for SEA. SEA was tested on the IEEE 4-bus Feeder and the IEEE European LV Test Feeder. The results were more accurate than the PFA results.

7. CONCLUSIONS

In conclusion, as responsible for the high data availability scenario, CERTH developed two algorithms with high utility, the PFA and SEA, each one of which also contained an additional important feature. For the PFA, a tap optimization algorithm was added to it, while for the SEA, a bad data detection algorithm was incorporated into it. The two main algorithms along with these features tackle most of the tentative use cases mentioned in Task 3.1 and will be implemented as a software tool in Task 3.5.

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9. ANNEX

9.1. CREATION OF THE h FUNCTIONS AND CONSTRAINTS FOR THE STATE ESTIMATION ALGORITHM

The possible measurements available for the SEA are voltage magnitudes, real and reactive power flows and real and reactive power injections. The subscript $\mathrm{ph} \in \{a, b, c\}$, denotes the phase of the measurement.

- Voltage magnitude measurements
  If the measurement of the magnitude of phase ph of bus i $V_{i,ph}$ is available, then
  $$h(\mathbf{x}) = V_{i,ph}$$

- Real and reactive power flow measurements
  If the measurement of the real or reactive power flow of phase ph of distribution line ij is available, then, depending on the type of power flow and its direction, h is chosen from the following:

$$h(\mathbf{x}) = P_{ij,a} = V_1 * \sum_{k=1}^{6} V_k * [G_{1k} * \cos(\delta_1 - \delta_k) + B_{1k} * \sin(\delta_1 - \delta_k)]$$

$$h(\mathbf{x}) = Q_{ij,a} = V_1 * \sum_{k=1}^{6} V_k * [G_{1k} * \sin(\delta_1 - \delta_k) - B_{1k} * \cos(\delta_1 - \delta_k)]$$

$$h(\mathbf{x}) = P_{ij,b} = V_2 * \sum_{k=1}^{6} V_k * [G_{2k} * \cos(\delta_2 - \delta_k) + B_{2k} * \sin(\delta_2 - \delta_k)]$$

$$h(\mathbf{x}) = Q_{ij,b} = V_2 * \sum_{k=1}^{6} V_k * [G_{2k} * \sin(\delta_2 - \delta_k) - B_{2k} * \cos(\delta_2 - \delta_k)]$$

$$h(\mathbf{x}) = P_{ij,c} = V_3 * \sum_{k=1}^{6} V_k * [G_{3k} * \cos(\delta_3 - \delta_k) + B_{3k} * \sin(\delta_3 - \delta_k)]$$

$$h(\mathbf{x}) = Q_{ij,c} = V_3 * \sum_{k=1}^{6} V_k * [G_{3k} * \sin(\delta_3 - \delta_k) - B_{3k} * \cos(\delta_3 - \delta_k)]$$

$$h(\mathbf{x}) = P_{ji,a} = V_4 * \sum_{k=1}^{6} V_k * [G_{4k} * \cos(\delta_4 - \delta_k) + B_{4k} * \sin(\delta_4 - \delta_k)]$$

$$h(\mathbf{x}) = Q_{ji,a} = V_4 * \sum_{k=1}^{6} V_k * [G_{4k} * \sin(\delta_4 - \delta_k) - B_{4k} * \cos(\delta_4 - \delta_k)]$$

$$h(\mathbf{x}) = P_{ji,b} = V_5 * \sum_{k=1}^{6} V_k * [G_{5k} * \cos(\delta_5 - \delta_k) + B_{5k} * \sin(\delta_5 - \delta_k)]$$

$$h(\mathbf{x}) = Q_{ji,b} = V_5 * \sum_{k=1}^{6} V_k * [G_{5k} * \sin(\delta_5 - \delta_k) - B_{5k} * \cos(\delta_5 - \delta_k)]$$

$$h(\mathbf{x}) = P_{ji,c} = V_6 * \sum_{k=1}^{6} V_k * [G_{6k} * \cos(\delta_6 - \delta_k) + B_{6k} * \sin(\delta_6 - \delta_k)]$$

$$h(\mathbf{x}) = Q_{ji,c} = V_6 * \sum_{k=1}^{6} V_k * [G_{6k} * \sin(\delta_6 - \delta_k) - B_{6k} * \cos(\delta_6 - \delta_k)]$$

where

$$V_1 = V_{i,a}$$
$$\delta_1 = \delta_{i,a}$$
$$V_2 = V_{i,b}$$
$$\delta_2 = \delta_{i,b}$$
$$V_3 = V_{i,c}$$
$$\delta_3 = \delta_{i,c}$$
$$V_4 = V_{j,a}$$
$$\delta_4 = \delta_{j,a}$$
$$V_5 = V_{j,b}$$
$$\delta_5 = \delta_{j,b}$$
$$V_6 = V_{j,c}$$
$$\delta_6 = \delta_{j,c}$$

$\overline{\mathbf{Y}} = \mathbf{G} + j * \mathbf{B}$ is the matrix of the transformer or distribution line, given in the respective SEA modelling section.

- Real and reactive power injection measurements
  If the measurement of the real or reactive power injection of phase ph of bus i is available, then h is the sum of ij flows from the previous section and the load components:

$$h(\mathbf{x}) = P_{i,ph} = \sum_{\text{for every Z load on bus } i} P_{Z,i,ph} + \sum_{\text{for every I load on bus } i} P_{I,i,ph} + \sum_{\text{for every bus j connected with bus i}} P_{ij,ph}$$

$$h(\mathbf{x}) = Q_{i,ph} = \sum_{\text{for every Z load on bus } i} Q_{Z,i,ph} + \sum_{\text{for every I load on bus } i} Q_{I,i,ph} + \sum_{\text{for every bus j connected with bus i}} Q_{ij,ph}$$

The load component for each load is chosen from the following:

- if a Z load is Y_g connected:

$$P_{Z,ph} = \frac{R_{ph} * V_{ph}^2}{R_{ph}^2 + X_{ph}^2}$$

$$Q_{Z,ph} = \frac{X_{ph} * V_{ph}^2}{R_{ph}^2 + X_{ph}^2}$$

$$R_{ph}, X_{ph} \text{ are known constants for each phase}$$

- if an I load is Y_g connected:

$$P_{I,ph} = V_{ph} * I_{ph} * \cos\theta_{ph}$$

$$Q_{I,ph} = V_{ph} * I_{ph} * \sin\theta_{ph}$$

$$I_{ph}, \theta_{ph} \text{ are known constants for each phase}$$

- Constraints

In addition to the given measurements of the power system, constraints must be included. These are the real and reactive power injections for each bus that has no PQ loads and no generators. For a bus i with no PQ loads and no generators, both real and reactive power injections are zero and thus:

$$0 = \sum_{\text{for every Z load on bus } i} P_{Z,i,ph} + \sum_{\text{for every I load on bus } i} P_{I,i,ph} + \sum_{\text{for every bus j connected with bus i}} P_{ij,ph}$$

$$0 = \sum_{\text{for every Z load on bus } i} Q_{Z,i,ph} + \sum_{\text{for every I load on bus } i} Q_{I,i,ph} + \sum_{\text{for every bus j connected with bus i}} Q_{ij,ph}$$

So, aside from the given measurements, 2*b additional constraints must be considered, where b is the number of buses with no PQ loads and no generators.

Additional constraints may be given in the form of inequalities. Such constraints may be upper and lower bound voltage magnitudes on some buses:

$$V_{min} \leq V_{bus} \leq V_{max}$$

and upper and lower bound real and reactive power flows on some lines:

$$P_{min} \leq P_{line} \leq P_{max}$$

$$Q_{min} \leq Q_{line} \leq Q_{max}$$

9.2. CALCULATION OF THE LOAD POWER PER PHASE AFTER THE ESTIMATION OF VOLTAGES

The real and reactive power consumed at phase $ph \in \{a, b, c\}$, by loads of all kinds of a bus i with no generators are:

$$P_{L,i,ph} = -\sum_{\text{for every bus j connected with bus i}} P_{ij,ph}$$

$$Q_{L,i,ph} = -\sum_{\text{for every bus j connected with bus i}} Q_{ij,ph}$$

where $P_{ij,ph}, Q_{ij,ph}$ can be found from the h functions of section 7.1. The complex power is given by:

$$\bar{\mathbf{S}}_{\mathbf{L,i,ph}} = P_{L,i,ph} + j * Q_{L,i,ph}$$